LTI MODEL ORDER REDUCTION FOR FINITE SEGMENTS OF IMPULSE RESPONSE

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OUTLINE

the basic task (informal)
standard techniques applied to the basic task (overview)
maximal real part optimal model reduction
the formal model reduction setup and its optimal solution
numerical examples

SPECTRAL ANALYSIS OF LIGHTLY DAMPED OSCILLATIONS

given $h = \{h_t\}_{t=1}^{n}$ (noisy samples of of a superposition of lightly dampled oscillations), $m \ll m$
find a good approximation

$$\text{dist}\left(\{h_t\}, \left\{\sum_{i=1}^{m} A_i \rho_i^t \cos(\omega_i t + \phi_i)\right\}\right) \to \min$$

equivalently $\text{dist}(H(z), \hat{H}(z)) \to \min$ subject to an order constraint

MOMENTS MATCHING

find the $2m$ free coefficients of p, q from the set of $2m$ (linear) equations $h_t = \hat{h}_t$ for $t = 1, 2, \dots, 2m$

potentially, fitting the noise
some degrees of freedom to be used on stability
not likely to be close to optimality

the number of data points should be much larger than the number of decision parameters

LEAST SQUARES

find p, q by minimizing the sum of squares of the coefficients of

$$H(z)q(z) - p(z) = \sum_{t=-m+1}^{n} e_t z^{-t},$$

subject to $q(1) = 1$.

minimizing $Hp - q$ instead of $H - p/q$

not good when q is comes out with zeros $|z| \approx 1$

caring for stability destroys least squares simplicity

POD

POD corresponds to minimizing $\|Hp - q\|_2$ subject to a unit length constraint imposed on (p, q)

same complaints

ERROR NORM MINIMIZATION

Would be ideal:

$$\|H - \widehat{H}\| \to \min$$

for some reasonable norm $\|\cdot\|$

to the best knowledge of the author, there is no norm $\|\cdot\|$ on $\mathbf{R}^n$ for which the task has a polynomial time solution algorithm

HANKEL OPTIMAL MODEL REDUCTION

for infinite impulse responses of rational transfer functions, Hankel norm of the error can be optimized by a polynomial time algorithm

Hankel norm = L-Infinity (frequency domain) distance to the set of anti-causal impulse responses

provides a lower bound in H-Infinity MOR

actual H-Infinity error norm close to optimal

need complete information

cubic growth

MAXIMAL REAL PART MODEL REDUCTION

a good alternative to Hankel norm:

$$\|\Delta\|_{mrp} = \|\mathrm{Re}(\Delta)\|_\infty = \max_{|z|=1} |\mathrm{Re}(\Delta(z))|$$

minimized by a polynomial time algorithm
accepts passivity and real part interpolation
can be applied to frequency samples

SAMPLED MAX REAL PART MOR

$$\max_i |\text{Re}(H(j\omega_i) - \hat{H}(j\omega_i))| \to \min$$

if necessary, subject to

$$\text{Re}(\hat{H}(j\omega)) \geq 0, \ \text{Re}\ \hat{H}(j\theta_i) = \text{Re}\ H(j\theta_i).$$

STRONG MAX REAL PART MOR

optimize in polynomial time

$$\|H - \hat{H} - V\|_\infty \to \text{min},$$

where $\hat{H}, V$ are constrained by

$$\hat{H}(z) = \frac{p(z)}{q(z)}, \quad V(z) = \frac{r(z)}{z^m q(1/z)},$$

and q is a Schur polynomial of order m

also with passivity and interpolation

STRONG MAX REAL PART MOR

better lower bound than in Hankel MOR

also the sampled version

$$\max_i |H(z_i) - \hat{H}(z_i) - V(z_i)| \to \min.$$

with passivity and interpolation constraints

EXPLANATION

one-to-one correspondence

$$\hat{H} + V = \frac{b + jc}{a}$$

convexification of

$$\left| \frac{b + jc}{a} - H \right| \le \gamma, \quad \gamma \to \text{min},$$

into

$$|b + jc - Ha| \le \gamma a, \quad \gamma \to \text{min}$$

MAIN RESULT

polynomial time algorithm for

$$E(\hat{h}, v) = \|h - \hat{h} - v\|_{n,\infty} \to \text{min},$$

where

$$\frac{p(z)}{q(z)} = \sum_{t=0}^{\infty} \hat{h}_t z^{-t} \quad (|z| \geq 1),$$

$$\frac{r(z)}{z^m q(1/z)} = \sum_{t=0}^{\infty} v_{n+1-t} z^t \quad (|z| \leq 1).$$

EXPLANATION

one-to-one correspondence

$$P_n(p/q) = p_1/q_1 \quad \text{at} \quad z = \exp\left(\frac{2\pi kj}{n+1}\right)$$

FMR APPLICATIONS

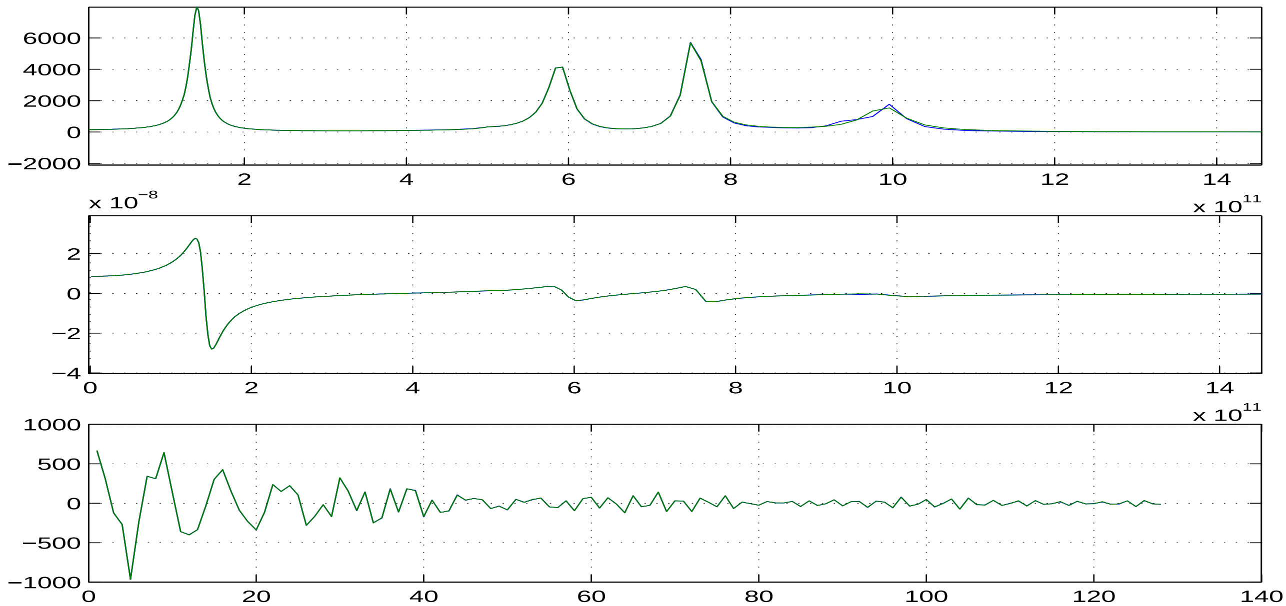
given $G(s) = C(sE - A)^{-1}B$, calculate

$$g_k : \; H(z) = G\left(\omega_0 \frac{z-1}{z+1}\right) = \sum_{k=0}^{\infty} h_k z^{-k}$$

complexity: solving equations $(\omega_0 E - A)x = z$ (sometimes can be reduced to the case $Mx = \tilde{z}$ with $M = M'$)

$\hat{H}(z)$ obtained via max real MOR on $\{h_k\}_{k=0}^{n}$

RF INDUCTOR (16/128)



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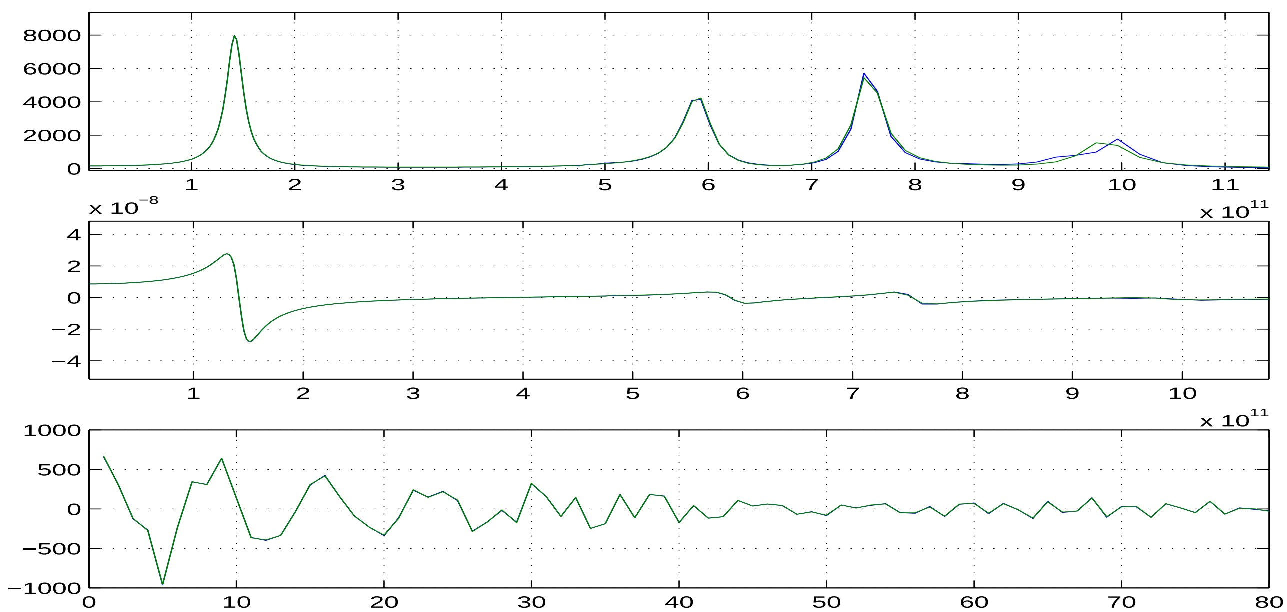

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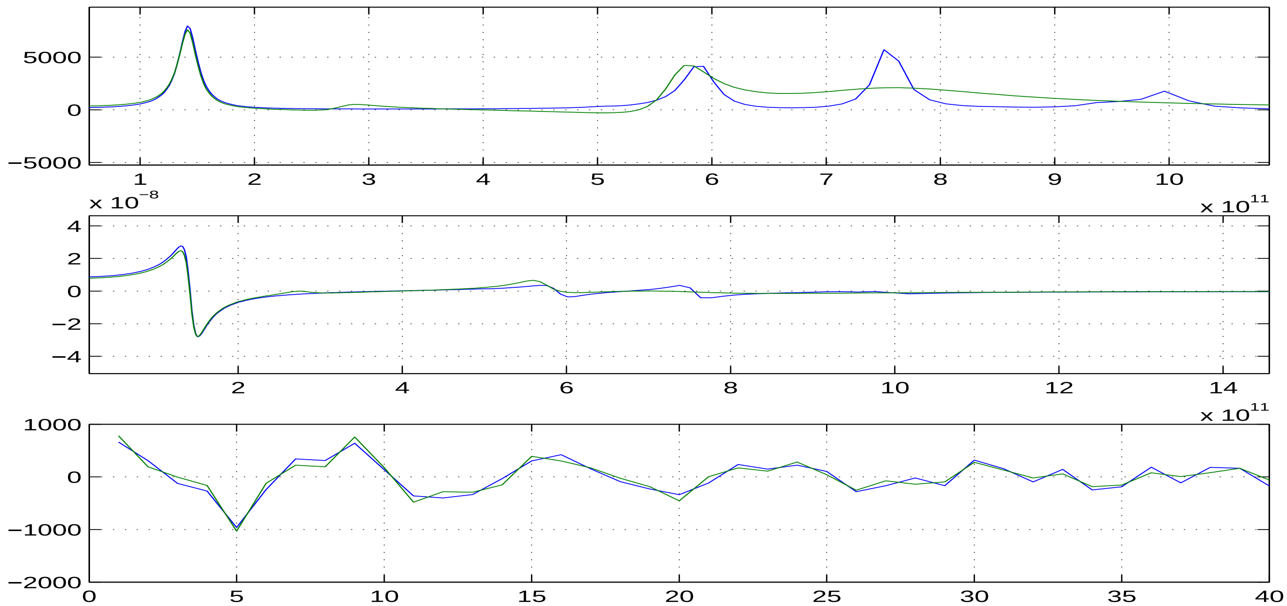
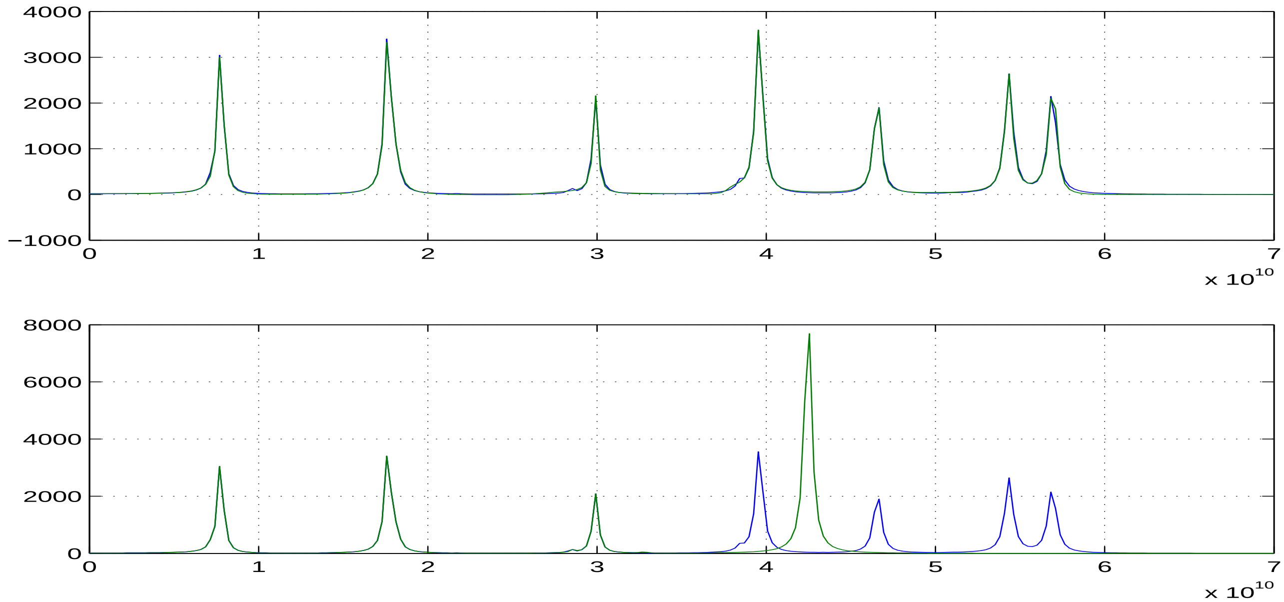
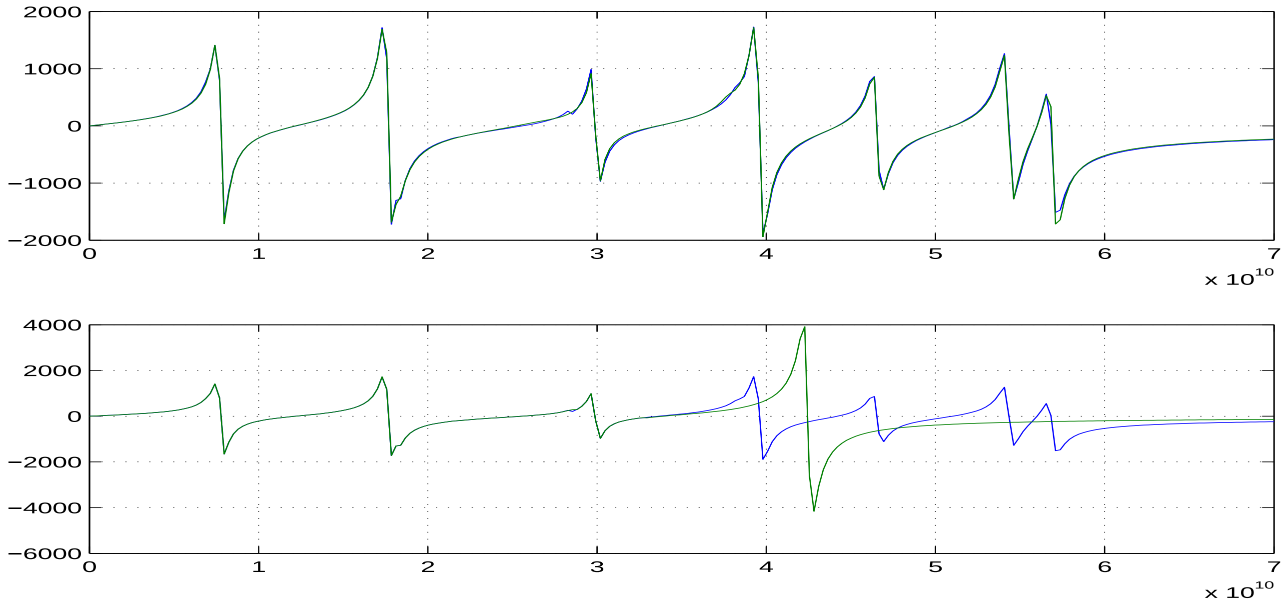


POWER GRID (20/512) REAL PART



POWER GRID (20/512) IMAGINARY PART



POWER GRID (20/512) FOURIER COEFFICIENTS